Experimental Modal Analysis of Reinforced Concrete Girder using Appropriated Excitation Technique

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Abstract-Vibration based structural health monitoring of civil infrastructure is becoming very popular due to advancement in instrumentation and development of more robust and powerful system identification techniques. Damage alters the dynamic characteristics of a structure and this relation is used to identify, locate and assess the severity of damage. Despite of advances in vibration based methods very limited success has been reported for reinforced concrete structures particularly in field applications due to complexity of civil engineering structures, limited measurement points, measurement noise and processing errors. The success of vibration based methods relies on the ability to precisely measure the modal properties. Experimental modal analysis is often carried out to observe the modal parameters among which phase separation methods are very common. Phase resonance methods traditionally used in the field of aerospace and mechanical engineering measures the modal parameters physically rather than mathematically. This paper presents the methodology of phase resonance method with application to a typical reinforced concrete bridge girder reduced to one-fourth scale. Piezoelectric accelerometers at 54 measurement points are used to obtain five modal parameters i.e. natural frequency, mode shapes, damping ratio, generalized mass and stiffness. The comparison shows noticeable variation in extracted modal parameters.

Keywords-Vibration Based Damage Detection, Experimental Modal Analysis, Reinforced Concrete, Force Appropriation Method, Modal Parameters

I. INTRODUCTION

Civil infrastructure is exposed to aggressive environment, ever increasing traffic volume, various man-made and natural hazards which might lead to severe damage or cascading catastrophic failure. To minimize and reduce these risks, prompt and intensive monitoring is required. With the aim to diagnose the structural damages at the earliest possible stage and to evaluate the remaining useful life referred as prognosis, the concept of Structural Health Monitoring (SHM) has received increasing attention among civil engineering research community [i]. Current bridge inspection and evaluation mostly rely on visual inspection and Non Destructive Methods with their known limitations and ability to check the structure locally. The progress on new and emerging sensory technologies, sophisticated data acquisition systems and automated analysis tools has led to development of global automated structural integrity assessment techniques.

In the past few decades, significant amount of research work has been carried out on non-destructive damage detection of structure using changes in modal parameters. The extension of Vibration Based Damage Detection (VBDD) techniques for SHM of civil infrastructure is progressively gaining attention among researchers [ii]. The basis of VBDD methods is that structural modal parameters (natural frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Damage will affect the physical properties of a system, which consequently will change the measured dynamic response of the system [iii]. Despite of advances in VBDD methods, very limited progress has been reported in field applications mainly because of errors encountered due to limited measurement points, noise, processing error and complex post cracking behavior of reinforced concrete structures. Therefore, much less research and development work has been reported for damage detection of reinforced concrete structures [iv]. It is necessary to investigate these effects to develop robust and reliable procedures for accurate and advanced methods to locate and quantify damage in reinforced concrete structures.

II. LITERATURE REVIEW

Vibration-based structural health monitoring has been subject of great interest among the field of civil and mechanical engineering communities since last two decades. Various methodologies and damage detection techniques have been comprehensively summarized by References [v-vii]. A common approach is to measure the vibration response and compare the damage-sensitive parameters to distinguish between damaged and undamaged state. This process can be categorized into forward problem and inverse problem. In forward problem, damages are induced with known properties and their corresponding effects on modal properties are studied whereas the inverse problems are addressed with inverse approach. Reference [viii] presented an overview of inverse methods used for damage detection from measured vibration data. According to [ix] presented a review on the utilization of changes in resonant frequency to detect structural damage. Reference [x] studied the development in mode shape based structural damage identification methods. The application of vibration-based methods with special emphasis on composite materials has been described by According to [xi]. The success of any vibration based structural health monitoring technique relies on the ability to precisely measure the modal properties.

Modal parameters may be obtained analytically using finite element method (FEM) or experimentally through different experimental techniques. Normal Mode Analysis (NMA) method, also called as Phase Resonance method, has been traditionally used in aerospace industry for ground vibration testing of aircrafts [xii]. Phase resonance method was used for Experimental Modal Analysis (EMA) to identify modes and modal parameters until the advent of Fast Fourier Transform (FFT), which gave rise to the evolution of the broadband test technique (phase separation). Subsequently Multi-Degree of Freedom (MDOF) curve fitting techniques were developed to identify the modal properties from multiple Frequency response functions (FRFs). These two methods are explained in Section III.

The accurate prediction of modal properties is entirely dependent on the assumption of linearity which however cannot be justified for composite complex structures, resultantly the FRFs at operational vibration levels differ from those observed at low excitation level. As non-linearity is a major concern in phase separation method whereas in case of phase resonance, it is possible to characterize the non-linear modal behavior by iteratively varying the inputs and obtaining the output consistent with the operating conditions.

Further, the development of new display indicators such as multi Lissajous displays and automation techniques has reduced the time taken to tune each mode using multiple shakers. The combination of these two factors has revived interests in the phase resonance methods. Reference [xiii] discussed application of phase separation and phase resonance methods to both proportionally and non-proportionally damped structures with their application to an aluminum plate. For non-proportional damped structures exhibiting significant modal overlapping and in cases where close correlation with Finite Element Analysis is required, the phase resonance method employing force appropriation technique has been proved valuable.

The EMA of Reinforced Concrete (RC) structures through phase resonance method has not been presented so far. With this motivation, the author carried out EMA of reduced scale bridge girder (1/4th) using force appropriation method for identification of natural frequency, mode shapes, damping, generalized mass and generalized stiffness. These five modal parameters were observed for both undamaged and damaged case and comparison of results shows considerable variation in observed modal parameters which can be further used for damage detection and localization purpose. The results obtained are discussed in Section V.

III. EXPERIMENTAL MODAL ANALYSIS

Modal parameters estimation is carried out through modal analysis performed by calculation (Finite Element Analysis, FEA) or by testing employing various methods of experimental modal testing broadly classified in two groups depending on the nature of excitation method:

Global broad-band excitation (phase separation)

Appropriated excitation (phase resonance)

The global excitation method excites all desired modes simultaneously using single or multiple broad-band excitation signals. The mode separation is carried out mathematically by applying various curve-fitting algorithms on set of measured FRF. Modal parameter estimation algorithms are used to extract mode data from each separated measured mode.

Force Appropriated Modal Testing (FAMT) also known as Normal Mode Analysis (NMA) or Phase Resonance method achieve modal separation physically rather than mathematically. The structure is excited physically by applying coherent sinusoidal vibration at a resonant frequency forcing a single mode of vibration to be sustained at a time. The excitation force distribution is tuned (automatically adjusts the amplitude and phase of the forces) to achieve mode isolation by ensuring that all velocity responses are exactly either in-phase or out-of-phase with the applied forces. Each mode is studied in isolation from all other modes in its purest possible form to obtain mode shape from array of measured responses. The generalized parameters are extracted by observing the effect of induced additional quadrature force.

A typical force-apportioned normal mode test involves following steps

The first step is to identify the number of modes and the approximate natural frequencies by applying sine sweep/random excitation across the entire frequency bandwidth.

In the second step, each mode is isolated with appropriate tuning of the level of force and frequency of excitation. The number and location of sensors and shakers should be well optimized. This iterative tuning is aided by two global indicators i.e. the Modal Indicator Function (MIF) approaches unity while the Phase Indicator (PI) approaches zero. The tuning of both the force and phase at the excitation point may be controlled by a closed loop. A stabilization time should be allowed at each iteration for the responses to settle.

Once a mode is isolated and sustained by the appropriate forces, responses at all sensors are measured simultaneously ensuring that all the responses are in mono-phase and in phase quadrature with the excitation force vector. The mode shape may be obtained directly from array of measured response amplitudes while maintaining the structure in brief resonant dwell.

The calculations of generalized mass and stiffness are made using the Complex Power (CP) and Quadrature Force (QF) methods and damping parameters by applying frequency sweep around resonances. The CP function is derived from the summation of real and imaginary components of all drive-site-force-velocity products during a narrow frequency sweep across the natural frequency. The real or active part of CP reaches maximum and the imaginary part crosses zero at the natural frequency. The generalized parameters are extracted from the SDOF curve fit of the complex power spectrum.

Normal mode testing uses sine excitation with multiple shakers in a Multiple Input Multiple Output (MIMO) arrangement which offers an intuitive way of modal analysis with following advantages over other techniques.

In order to get sufficient vibration energy in large and complex structures using single shaker, there is a tendency to overdrive the excitation DOF which often results in nonlinear behavior and deteriorates the FRF estimation. In case of MIMO, the input force energy is distributed over more locations on the structure providing a more uniform vibration response over the structure.

Multiple location excitations also provide a better simulation of structures in real life operations.

Normal mode shapes can be obtained directly without utilizing Frequency Response Function (FRF) or modal parameter estimation techniques.

The structure is excited in a single mode by controlled force appropriation and enhances the comparison with FEA or other excitation methods.

The nonlinear behavior of structures can be accurately studied by amplitude sweeping.

The acquisition and processing of modal parameters is very fast so that test setup can be validated simultaneously.

IV. THEORETICAL BACKGROUND

Consider a linear time invariant system with n degrees of freedom (DOF) subject to sinusoidal excitation force vector $\{f\}$ at an angular frequency ω. The n second order linear differentials equations with constant coefficients to describe the force-response relationship of the system may be written in matrix format as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{1}$$

The M, C and K matrices represent mass, damping and stiffness respectively. In general, these n equations are coupled so that these matrices have non-zero off-diagonal elements. These equations are uncoupled by normal mode testing for identification of mode shapes. The complex displacement response vector of the structure can be described as

$$\{x\} = [A(\omega) + iB(\omega)]\{f\} \tag{2}$$

Where $[A]$ and $[B]$ denotes the real and imaginary parts of the FRF matrix respectively. The phase resonance condition is said to be achieved when the response of the structure is in mono-phase (0 or 180° phase) and in quadrature (90° phase) with the sinusoidal excitation so that the structure is exciting in its i^{th} undamped normal mode at corresponding undamped natural frequency (ω_i). At this condition, the excitation force vector will be real, imaginary part of the response vector will correspond to undamped normal mode shape and real part will be zero, i.e.

$$Imaginary\ \{f\}_i = 0 \tag{3}$$

$$Real\ \{x\} = [A]\{f\}_i = 0 \tag{4}$$

$$Imaginary\ \{x\} = [B]\{f\}_i = \{\emptyset\}_i \tag{5}$$

Where $\{f\}_i$ is the appropriated force vector of the i^{th} mode shape $\{\phi\}_i$. The condition often holds for structures with low damping and sufficiently separated natural frequencies or when response of the structure is dominated by one specific natural frequency at the appropriated excitation force vector. The vibrating multiple DOF system can be described as generalized single DOF system at corresponding natural frequency as

$$\omega_r{}^2 = \frac{k_r}{m_r} \tag{6}$$

Where k_r and m_r are generalized stiffness and mass values, being global characterizing parameters for the individual eigenvectors respectively mode shapes. If the eigenvector Φ_r is normalized to its largest element, then these generalized properties can be calculated:

$$m_r = \Phi_r{}^T . M . \Phi_r \tag{7}$$

$$k_r = \Phi_r{}^T . K . \Phi_r \tag{8}$$

When the full mass matrix is replaced by a diagonal lumped mass matrix, the expression for the generalized mass becomes:

$$m_r = \sum(m_i . x_i{}^2) \tag{9}$$

Where xi are the elements of the normalized eigenvector Φ. The basis of appropriation method is empirical adjustment of inputs and evaluation of the quality of mode isolation. To evaluate the quality of mode isolation, the phase criterion and quality criterion are often used.

The phase criterion determines that phase of particular reference velocity sensor to be real (imaginary for acceleration or displacement). It gives a good approximation with low modal density i.e, other modes are not significantly coupled with the isolated mode. The quality criterion evaluates the quality of appropriation using the relative importance of measured real and imaginary responses information. Some of the quality criterion are described as under

The Multivariate Mode Indicator Function (MMIF) is the ratio of the quadrature energy to the total energy. The mass matrix is introduced to compare energies. The value of MMIF is 0 for a perfect appropriation.

$$q(s) = \frac{\{|Im\, \dot{x}|\}^T M \{|\dot{x}|\}}{\{|\dot{x}|\}^H M \{|\dot{x}|\}} \tag{10}$$

The statistical comparison between two mode shapes can be performed through Modal Assurance Criterion (MAC), which gives a correlation coefficient between two mode shapes. The coefficient value of 1.0 means that the two shapes are perfectly correlated. Practically, any value between 0.9 and 1.0 is considered good correlation. The MAC for two modes r and s can be calculated using following equation

$$MAC = \frac{\left|\{\emptyset_r\}^T \{\emptyset_s\}\right|^2}{(\{\emptyset_r\}^T \{\emptyset_s\})(\{\emptyset_s\}^T \{\emptyset_r\})} \tag{11}$$

V. EXPERIMENTAL INVESTIGATION

One RC girder of size 3660mm x 310 mm x 155 mm was cast with mix proportion of 1:1.25: 2.25: 0.45 (cement: Fine Aggregate: Coarse Aggregate: water cement ratio) which represents M35 grade of concrete. The Grade-60 reinforcement bars conforming to AASHT0 M31 Grade 60 steel specification were used. The geometry and cross section of the model is shown in Fig. 1. The total mass of the model is 510 kg which results in a density of $\rho = 2{,}900$ kg/m^3. The material properties are taken from actual girder. The design process of actual reinforced concrete girder followed the AASHTO LRFD bridge design specifications. The vehicle live loads were applied to produce maximum stresses as specified in West Pakistan Code of Practice for Highway Bridges 1967 (WPCHB) which includes 70 Ton tracked Military Vehicle and 54.5 Ton train of trailers. The impact loads and wind loads are taken in accordance with provision of WPCHB.

These loads were applied in combination with side walk live load of 5 KN/m^2, horizontal live load on side barriers according to Article 2.7 of AASHTO and other loads e.g. dead loads etc. The RC bridge was designed and dimension of actual girder becomes (14640 x 1240 x 620) mm in length, width and height respectively.

This model bridge girder has been scaled down to 1/4th of actual bridge girder. The dimensions and material properties of model girder are selected confirming the modeling and similitude requirements of replica scaling method. While casting the model girder, vibratory compaction of concrete was carried out to ensure removal of air and proper bond between concrete and steel to avoid slippage. Proper curing was carried out carefully.

A. Modal Testing

The model girder is mounted to vertical stands with the help of flexible bungees to simulate the free-free boundary conditions as shown in Fig. 4. The piezoelectric accelerometers having sensitivity of 6 pC/g and frequency range up to 3k Hz are calibrated and attached to model surface at 54 selected points with wax to measure the responses. The arrangement of accelerometers is shown in Fig. 1.

Electrodynamic shakers (also called modal exciters) are employed to provide a known input (force and frequency) to the structure. Working on the principle of electromagnetic induction, the electrodynamic exciter provides accurate control of the amplitude and phase of force which is very importantly required in force appropriation testing. Two exciters capable of exciting up to constant magnitude of 200 N are used with stingers as interface between exciter and the model girder. The purpose of stinger is to apply only axial forces with high fidelity and prevent lateral constraint forces and moments. The mass of the exciters is balanced through elastic cords. The exciters suspension system is used to minimize any influence of exciters mass on measurements. The experimental arrangement is shown in Fig. 4 &5.

The selection of exciters position is very critical and should be selected according to type of mode to be excited (i.e. symmetric or anti-symmetric mode). The exciters are attached at extremities of free-free model beam in a symmetrical axis for extraction of symmetrical modes and inversely arranged for anti-symmetrical modes not only to excite modes of interest but also to cancel the contribution of off resonant modes.

The excitation is controlled by P-SYS Modal software, a Dynamic Signal Analyzer transmits the signals through an amplifier which is transformed to physical excitation by electromagnetic exciters. P-Win Modal software is used to analyze the responses

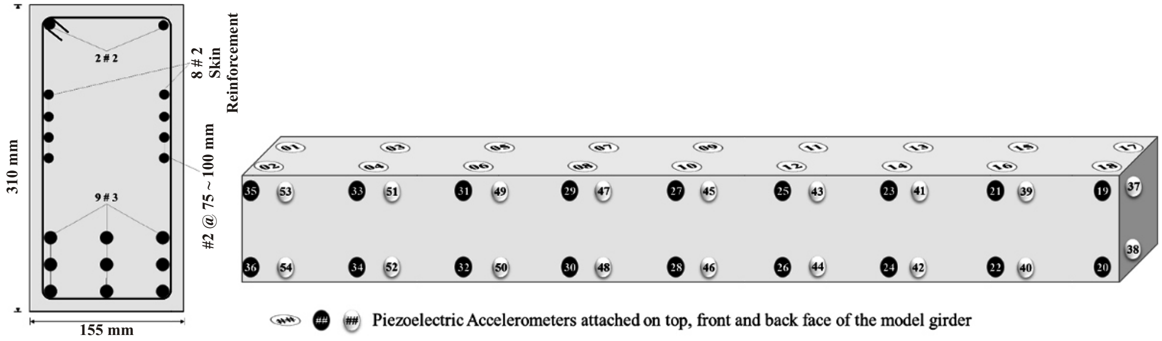

Fig. 1. Model girder cross-section and Piezoelectric Accelerometers arrangement

measured through accelerometers and compute the FRF for modal analysis. The natural frequencies, damping ratios and mode shapes are obtained through Complex Power Method.

Natural frequencies of firstly identified through an impact and then by sweeping slowly within across the identified natural frequency. The analysis found seven physical modes within a frequency range of 0 to 350 Hz.

B. Static Flexure Test

After the experimental modal analysis of undamaged model girder, the five point static flexure test was performed on the model to introduce the damages. The loading was applied with three actuators of 8 Ton maximum capacity and were equally spaced at a distance of 600mm between point loads and support span was 3300mm as shown in Fig. 2. Three Linear Variable Differential Transducer (LVDT) were fixed as shown in Fig. 2 to record the deflection at mid span as well as at a distance of L/3 from extreme ends where L is the total beam length. The maximum combined load of 24 Ton was applied with corresponding maximum deflections of 17.76mm recorded.

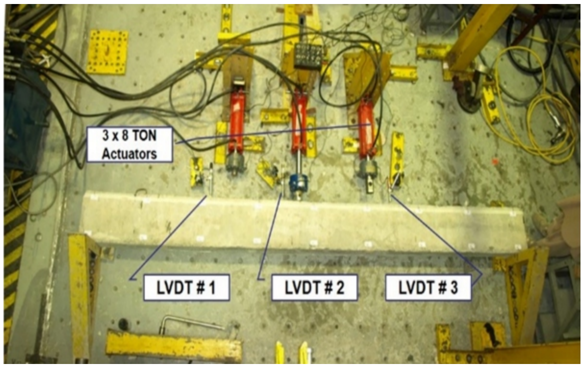

Fig. 2. Static Flexure Test arrangement

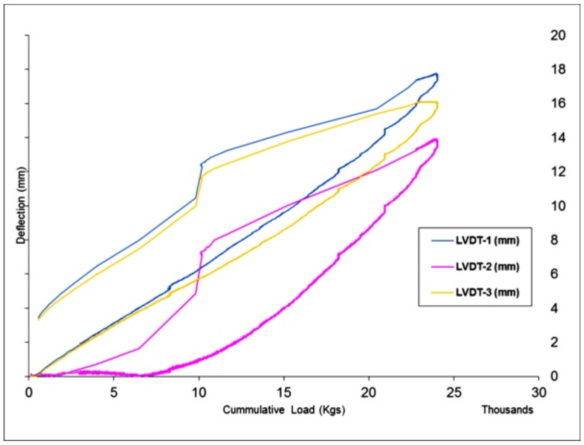

Fig. 3. Load vs Deflection plot of Static Flexure Test

Fig. 4. Experimental Modal Testing arrangement with exciter support system

Fig. 5. Experimental Modal Testing arrangement with free-free boundary conditions

The cracks pattern is studied by visual inspection. After the application of loading through static flexure test, the modal analysis of model girder in damaged state was again carried out with the similar arrangement. In the force appropriation method, a narrow frequency sweep across the resonant frequency and summation all drive-site force velocity products for each mode gives a CP spectrum.

TABLE 1
DETAILS OF FIRST SEVEN MODES IDENTIFIED

Mode	Description	Axis	Amplitude of Ref Transducer x 10^{-6} (m)		Exciter Position	
			Undamaged	Damaged	(Transducer No.)	
1	First Symmetric Bend	Y	255.45	254.63	36	20
2	First Symmetric Bend	Z	25.44	21.59	2	18
3	First Anti-Symmetric Bend	Z	5.92	4.83	2	18
4	First Anti-Symmetric Bend	Y	23.27	19.91	35	19
5	Symmetric Second Bend	Y	7.19	6.36	35	19
6	Twist	-	4.81	4.02	36	19
7	Anti-Symmetric Second Bend	Y	1.00	1.95	35	19

The real part (active power/in-phase with force) and the imaginary part (reactive power/quadrature with force) of first four mode shapes of the model girder in damaged and undamaged state are shown in figures 6 to 9 When the mode is perfectly tuned, the active component should be maximum and reactive component should be zero

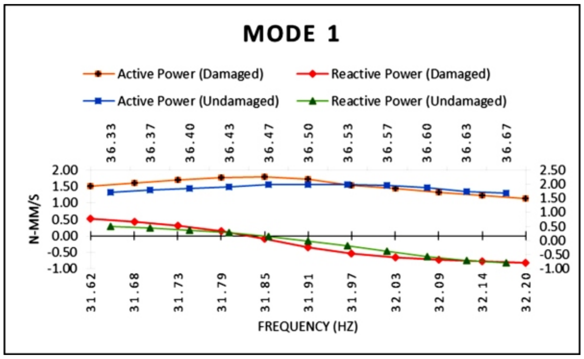

Fig. 6. Complex Power Spectrum for isolation of Mode 1

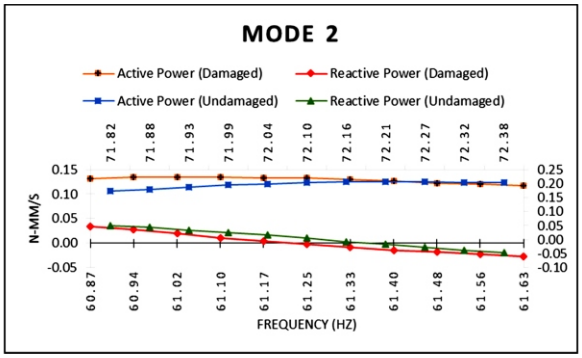

Fig. 7. Complex Power Spectrum for isolation of Mode 2

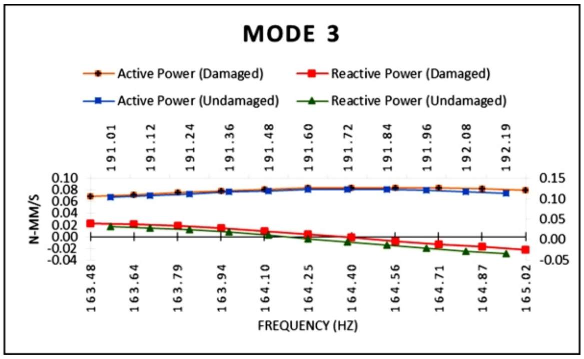

Fig. 8. Complex Power Spectrum for isolation of Mode 3

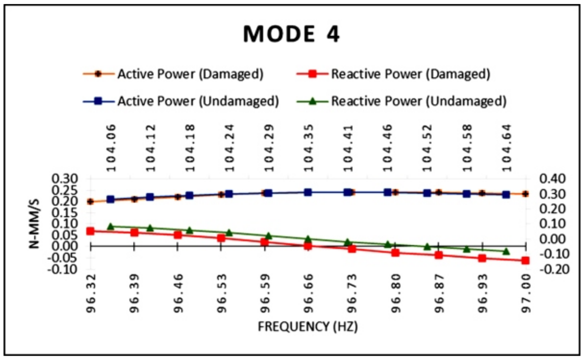

Fig. 9. Complex Power Spectrum for isolation of Mode 4

VI. RESULTS AND DISCUSSION

A Natural Frequency

A structure subjected to certain degree of damage experiences a change in stiffness [xiv] which subsequently causes its natural frequency to change. The magnitude of the frequency shift is also an indicator of the damage severity. The results are compared which shows the reduction in natural frequency of damaged model girder in comparison with undamaged case. The magnitude of force applied simultaneously by exciters is also presented in Fig. 10.

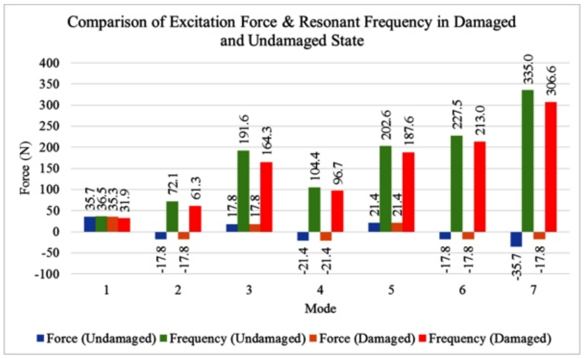

Fig. 10. Comparison of Resonant Frequency

B. Modal Damping

Modal damping of concrete member is associated with its dynamic response and can be used as a means to assess material integrity or damage occurred to the structure. Damping affects the rate at which vibrations decay and the response of the structures at resonance. The values of modal damping ratio observed for the undamaged and damaged beam are presented in Fig. 11. The results shows that damping ratios increases for the damaged beam, similar trend was observed by the author [xv, xvi].

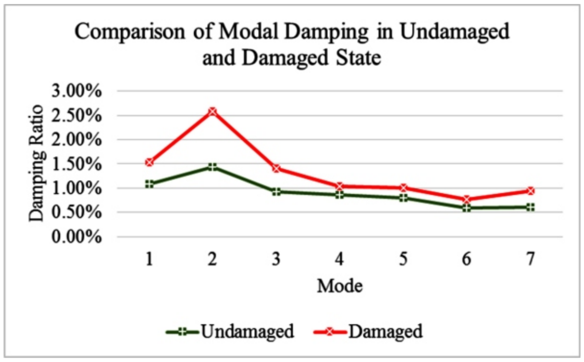

Fig. 11. Comparison of Modal Damping

C. Normalized Generalized Mass and Stiffness

The generalized mass and stiffness of each mode can be evaluated from the total driving power required to maintain the structure in resonant dwell. The calculation of generalized mass and stiffness follows complex power (CP) method. The CP function is derived by adding the real and imaginary components of force velocity products at all shaker locations. The excitation frequency is varied over a small range across the approximately identified natural frequency. The real or active part of CP reaches a maximum whereas the imaginary or reactive part crosses zero at the natural frequency. The resulted spectrum is curve-fit to obtain the generalized properties. The results obtained are shown in Fig. 12& 13.

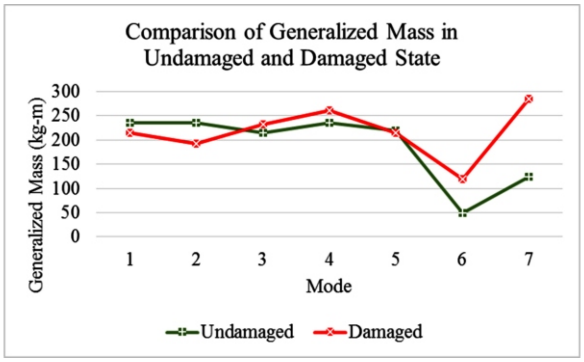

Fig. 12. Comparison of Generalized Mass

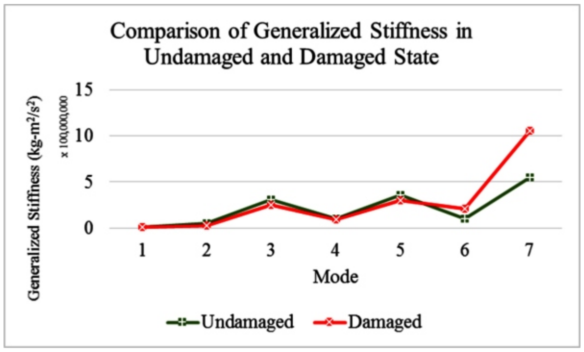

Fig. 13. Comparison of Generalized Stiffness

D. Mode Shapes

Modes are inherent properties of a structure and represents pattern of vibration executed by a structure at a particular frequency. Mode shapes are greatly affected by material and structural flaws because it changes the modal parameters. The normalized response of the measurement point in the form of mode shapes at a particular frequency and damping ratio is presented graphically in wireframe shapes (Fig. 14-27).

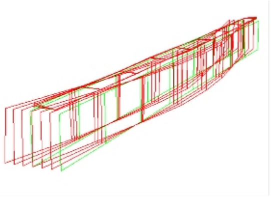

Fig. 14. Mode 1 Undamaged
36.5 Hz, 1.08%

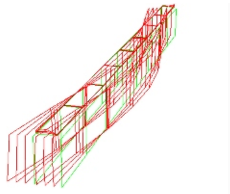

Fig. 15. Mode 1 Damaged
31.90 Hz, 1.53%

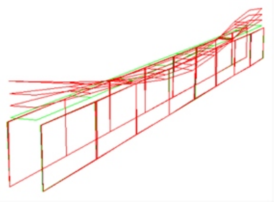

Fig. 16. Mode 2 Undamaged
72.1 Hz, 1.43%

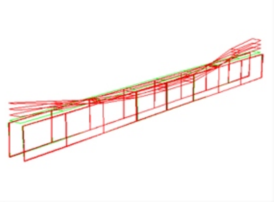

Fig. 17. Mode 2 Damaged
61.250 Hz, 2.58%

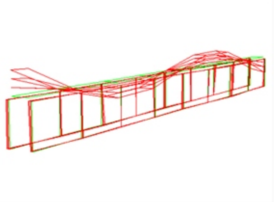

Fig. 18. Mode 3 Undamaged
191.6 Hz, 0.93%

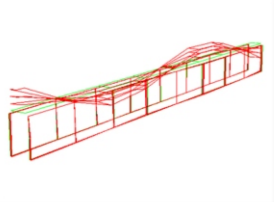

Fig. 19. Mode 3 Damaged
164.250 Hz, 1.40%

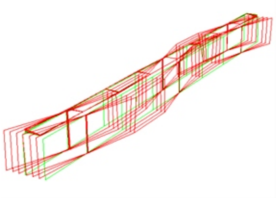

Fig. 20. Mode 4 Undamaged
104.35 Hz, 0.86%

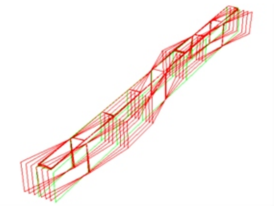

Fig. 21. Mode 4 Damaged
96.662 Hz, 1.04%

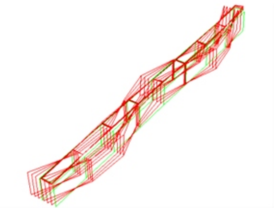

Fig. 22. Mode 5 Undamaged
202.60 Hz, 0.80%

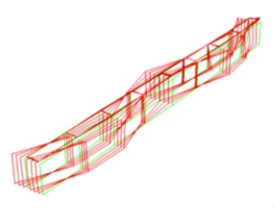

Fig. 23. Mode 5 Damaged
187.6 Hz, 1.00%

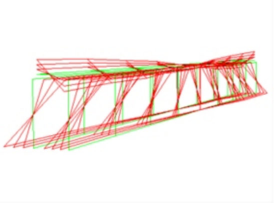

Fig. 24. Mode 6 Undamaged
227.45 Hz, 0.59%

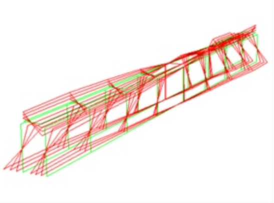

Fig. 25. Mode 6 Damaged
212.975 Hz, 0.76%

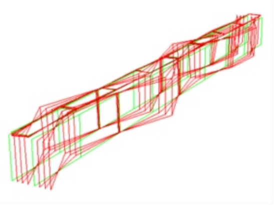

Fig. 26. Mode 7 Undamaged
335.00 Hz, 0.61%

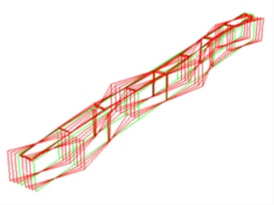

Fig. 27. Mode 7 Damaged
306.55 Hz, 0.94%

VII. CONCLUSION

Modal parameters are greatly affected by change in material properties or structural damages. The experimental modal analysis is performed on a RC beam in undamaged and damaged state with MIMO testing arrangement in a free-free boundary condition. Five modal parameters are observed and first seven modes are recorded with frequency range of 0 to 350 Hz. Damage was introduced thorough application of point load in 5 point static flexure test in which maximum deflection of 24mm at mid span is recorded.

The data is recorded through piezoelectric sensors installed at 54 measurement points in two lines on front side, back side and top. Two 200 N capacity electromagnetic exciters at extreme ends to free-free beam are installed. The modal analysis is carried out using force quadrature method and input force is given through force appropriation method.

The investigation through experimental modal testing confirm the detection of damage while comparing the data of undamaged and damaged states and following conclusions are made:

The appropriate force normal mode test can identify the natural parameters of each mode in isolation and provides extremely precise mode shapes even when adjacent modes overlap in modal bandwidth.

The frequency shift is observed in a range of 6.36% to 15.05%. The frequency shift is very clear in lower modes and the value reduces as the mode order increases which shows that initial modes are more sensitive to damage.

The damping ratios increases with damage in a range of 0.17 to 1.15% for respective modes. The damping ratio of both undamaged and damaged beam reduces with increase in mode number. The increase in damping ratio for damaged beam comparing to undamaged beam also exhibit the same trend.

The values of damping ratio decrease with higher modes indicating that lower modes have the tendency to decay much earlier.

The damping ratio in anti-symmetric modes of damaged structure shows more increase which shows rapid decaying capability of anti-symmetric modes.

The generalized mass decreases for symmetric modes and increases for antisymmetric modes. The increase is very significant for torsional mode and higher frequency antisymmetric mode.

The generalized stiffness decreases in first five modes and increases in case of torsional mode and higher frequency antisymmetric mode.

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